Discrete Structures

Name: **Application No:**

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1. The number of edges in a forest on n vertices containing k trees is

   _____

   (Note: Forest is a graph, whose connected components are trees)

2. A graph is said to be acyclic if it contains no cycles. A bipartite graph is a graph in which the vertex set can be partitioned into two such that no two vertices within the same partition are adjacent. Which of the following statements is(are) true?

   ⬜ Every cycle graph with an even number of vertices is bipartite
   ⬜ Every bipartite graph is a connected acyclic graph
   ⬜ Every connected acyclic graph is a bipartite graph
   ⬜ All complete graphs on at least 5 vertices are **not** bipartite

3. A graph has 24 edges and the degree of each vertex is k, then which of the following is the possible number of vertices? Select the most appropriate answer(s) only.
   ⬜ 9 ⬜ 8 ⬜ 10 ⬜ 20
   What is the value of k for the selected number of vertices? Give a proper justification.

   Ans: k=

   Justification:

4. Consider the below First Order Logic with predicates:
   car(x): x is a car,
   train(x): x is a train,
   slower(x,y): x is slower than y

   Statement: $\exists x (car(x) \wedge \forall y (train(y) \rightarrow slower(x,y)))$ when translated to simple english is
   Ans:

   Note: Do not use ‘exists’ or ‘forall’ words in your translation.

5. Given a set {1, 2, . . . , 1000} of integers. How many are divisible by 3 or 5? **Justify**.

6. The maximum number of edges in a simple graph with 10 vertices and 4 connected components is
   Ans:

   Justification:

7. Three fair dice are thrown simultaneously. Find the probability that all three dice have the same number on the faces showing up is _____
   Justification:

8. A fair die with faces {1,2,3,4,5,6} is thrown repeatedly till ′4′ is observed for the first time. Let X denote the number of times the dice is thrown. The expected value of X is _____
   Justification:

9. In a housing society, half of the families have a single child per family, while the remaining half have two children per family. The probability that a child picked at random has a sibling is _____
   Justification:

10. Passengers try repeatedly to get a seat reservation in any train running between two stations until they are successful. If there is a 40% chance of getting a reservation in any attempt by a passenger, then the average number of attempts that passengers need to make to get a seat reserved is _____
    Justification: